d'où $F(x) = \dfrac{-1}{3(3x+2)} + \dfrac{1}{(3x+2)^2} + \dfrac{1}{12}$

Solution 7 :

$F'(x) = -a \sin x + 3\, b \cos^2 x\, (- \sin x)$

$= -a \sin x - 3b \sin x\, (1 - \sin^2 x)$

$= - (a + 3b) \sin x + 3b \sin^3 x$

F étant primitive de f sur $\mathbb{R}$ si $F' = f \Leftrightarrow$

$$\begin{cases} 3b = 6 \\ -a - 3b = 2 \end{cases} \Leftrightarrow \begin{cases} b = 2 \\ a = -8 \end{cases}$$

D'où $F(x) = -8 \cos x + 2 \cos^3 x$

Solution 8 :

$$f(x) = \frac{x^3 - 3x^2 + 5}{(x-2)^2}$$

1°/ $f(x) = \dfrac{(ax+b)(x-2)^2 + c}{(x-2)^2} = \dfrac{(ax+b)(x^2 - 4x + 4) + c}{(x-2)^2}$

$$f(x) = \frac{ax^3 - 4ax^2 + 4ax + bx^2 - 4bx + 4b + c}{(x-2)^2}$$

$$= \frac{ax^3 + (b-4a)x^2 + x(4a-4b) + 4b + c}{(x-2)^2}$$

Par identification :

$$\begin{cases} a = 1 \\ b - 4a = -3 \\ 4a - 4b = 0 \\ 4b + c = 5 \end{cases} \Leftrightarrow \begin{cases} a = 1 \\ b = 1 \\ c = 5 - 4 = 1 \end{cases}$$

$$f(x) = x + 1 + \frac{1}{(x-2)^2}$$